Homework 1. Linear Algebra. Spring 2022. Prof. Pineiro

Print Name: ______________________________

1. Solve the following systems by Gauss-Jordan elimination. Determine if we have infinitely many solutions, unique solutions or no solutions

   (a)
$$\begin{cases} 3x + y - z = 1 \\ x - 2y + z = -3 \\ -2x + 2y + z = 5 \end{cases}$$

   (b)
$$\begin{cases} 2x_1 + 3x_2 + x_3 - 4x_4 = -5 \\ -x_1 + x_2 + 2x_3 + x_4 = 3 \\ -3x_1 + 2x_2 + 2x_3 - x_4 = 2 \end{cases}$$

   (c)
$$\begin{cases} x_1 + x_2 + x_3 - 4x_4 = -5 \\ -x_1 + x_2 + 2x_3 + x_4 = 3 \\ x_3 - x_4 = 2 \end{cases}$$

2. Find the value(s) of k so that the following linear system is consistent (that is, has at least one solution)
$$3x + 5y = 4$$
$$9x + ky = -1$$

3. Given the following homogeneous system of equations:
$$\begin{cases} x + y + z = 0 \\ x + 2y + 3z = 0 \\ -x + 3y - z = 0 \end{cases}$$

   (a) Would it be possible for the system to be inconsistent?

   (b) Find the row reduced echelon form.

   (c) Find the solution.

4. Find (if possible) the inverse of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ using the process of row-reduction.

5. Find (if possible) the inverse of the matrix $A = \begin{pmatrix} 3 & 2 & 3 \\ -4 & -4 & 5 \\ 1 & 1 & 1 \end{pmatrix}$ using the process of row-reduction.

6. Given the matrices:

$$A = \begin{pmatrix} 4 & 1 \\ -3 & 0 \\ 3 & 5 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 4 & -1 \\ 1 & 2 & -1 \end{pmatrix}$$

   Find $A \cdot B$ and $B \cdot A$.

7. Check that the inverse you found in question (4) satisfy the equation:

$$A \cdot A^{-1} = A^{-1} \cdot A = I_3.$$